328. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

If $x^2+xy+y^2=3a^2$, find the maximum value of $bx+cy$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

$x^2+xy+y^2=3a^2$, $bx+cy$=maximum.

$$\therefore \frac{dx}{dy}=-\frac{x+2y}{2x+y}=-\frac{c}{b}, \text{ or } y=\frac{(2c-b)x}{2b-c}.$$

$$\therefore x^2=\frac{a^2(2b-c)^2}{b^2-bc+c^2}; \text{ and } x=\pm\frac{a(2b-c)}{\sqrt{(b^2-bc+c^2)}},\quad y=\pm\frac{a(2c-b)}{\sqrt{(b^2-bc+c^2)}}.$$

$$\therefore bx+cy=2a\sqrt{(b^2-bc+c^2)}=2a\sqrt{\frac{b^3+c^3}{b+c}} \text{ is a maximum.}$$

Also solved by F. L. Griffin and S. G. Barton.

329. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

Between the quantities a and b there are inserted n arithmetical and n harmonical means, and a series of n terms is formed by dividing each arithmetical by the corresponding harmonical mean. Show that the sum of the series is, $n\left[1+\frac{n+2}{n+1}\cdot\frac{(a-b)^2}{6ab}\right]$.

Solution by HOWARD C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Neb., and S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

The n arithmetic means between a and b are:

$$(1)\ \frac{b+na}{n+1},\ \frac{2b+(n-1)a}{n+1},\ \dots,\ \frac{rb+(n-r+1)a}{n+1},\ \dots$$

The n harmonic means between a and b are:

$$(2)\ \frac{ab(n+1)}{nb+a},\ \frac{ab(n+1)}{(n-1)b+2a},\ \dots,\ \frac{ab(n+1)}{(n-r+1)b+(r-1)a},\ \dots$$

Dividing the terms of (1) by the corresponding terms of (2),

$$\frac{(b+na)(a+nb)}{ab(n+1)^2},\ \frac{[2b+(n-1)a][(n-1)b+2a]}{ab(n+1)^2},\ \dots,$$